Hetrodyne Laser Tracking at High Doppler Rates

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ABSTRACT: In hetrodyne laser tracking of a spacecraft, the incoming laser signal may be significantly offset from the local laser reference, producing a high-frequency beat note that must be accurately counted to produce an accurate phase measurement. At high Doppler rates, one limitation to the accuracy of this measurement is the stability of the spacecraft frequency standard, or clock. However, if a secondary laser is used, locked to the primary laser but offset from it in frequency, then the beat note between the two lasers provides a built-in frequency reference. What is more, the delay line produced by the travel time of the tracking signal provides a stable self-comparison that steers the frequency reference or measures its drift so that its instability may be corrected for. The resulting noise in the link is only the residual laser phase jitter and the shot noise in the phase measurement.

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I. Background

In a recent paper (Reference 1) an algorithm was proposed that would allow Michelson-type interferometers with unequal arms to perform nearly as well as those with exactly equal arms. The interferometer setup is a hetrodyne system with independent readouts of phase at each point of the interferometer, as shown in Figure 1. Signals from two central lasers, labeled 1 and 2, are sent out along the two independent directions. Lasers at the two end points are simultaneously sending signals back along the same two arms. At each of the four points, the relative phase of the incoming signal is compared with that of a fraction of the outgoing signal to produce a hetrodyne phase readout in each arm. At the same time, the two central points are sending and receiving an auxiliary phase signal between them, so that their phases can be tied together. If the arms were equal, then the data from each arm would simply be differenced to cancel phase jitter in the central lasers and leave the relative armlength changes (the quantity that interferometers are supposed to measure) as the remaining detectable cause of phase changes in the difference data. The point of the algorithm described in Reference 1 is to show that, instead of simply differencing that data in the two arms, the data from one arm can first be used to characterize the phase jitter in the central lasers. This allows one to model the noise that is introduced into the differenced data because of the unequal arms, The result is that the accuracy of the interferometer is not compromised.

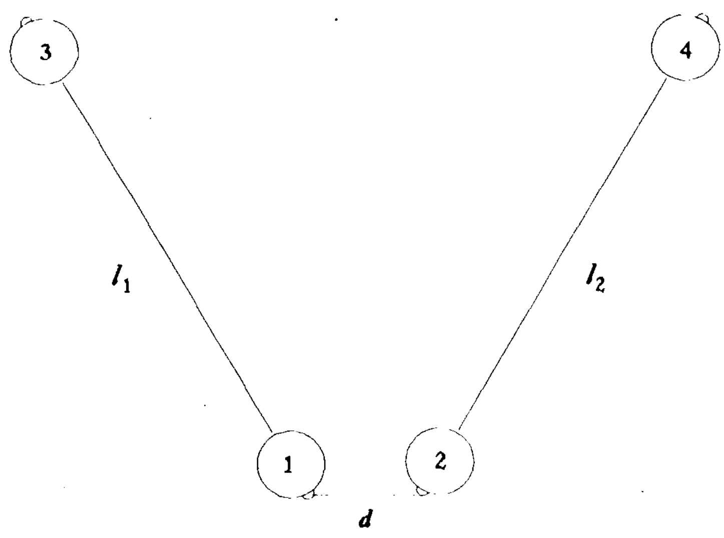

Figure 1. Geometry of the interferometer.

However, one limitation to this process that was not addressed in Reference 1 is a noise source that becomes important at high Doppler rates. When the incoming laser frequency is significantly different from the frequency of the on-board reference laser, a beat note at

RF will be created. in order to read out the high beat frequency with a small absolute phase error, a RF standard with good stability would be required. Unfortunately, for the Doppler rates to be expected in the gravitational wave experiment discussed in reference 1, the requirement on RF frequency stability is too stringent. It is the purpose of the present paper to describe a laser tracking system that provides its own RF standard and corrects for instabilities in this standard.

The rest of the paper proceeds as follows. In the next section we begin with a review of the unequal-arm algorithm, pointing out where the frequency standard instability creates the problem. Then, in Sections III and IV, a new laser transmitter and received are described which provide the correction procedure required. Finally, in Section V, the signal analysis procedure is described and the residual limitations of the new system are discussed.

II. Unequal-arm Interferometers

Each laser produces a signal

$$\phi_m(t) = \nu_m t + p_m(t),$$

where m goes from 1 to 4 and ν_m is the frequency and $p_m(t)$ the phase of the m^{th} laser. (Here and in future notation, all expressions such as $x(t)$ that can be read as functions are to be taken as functions.) The two central lasers of the interferometer send and receive local phase reference signals from each other, producing a received signal in each given by

$$\zeta_i(t) = \nu_j t - \nu_j d - \nu_i t + p_j(t - d) - p_i(t), \qquad \mathrm{II.1}$$

where $\{i, j\}$ are chosen from the set {1, 2} and d is the light time between the two spacecraft. in Reference 1, it is shown that a combination of ζ_1 and ζ_2 can be formed that will contain only the difference of the two laser phases

$$\sigma(t) = p_2(t) - p_1(t). \qquad \mathrm{II.2}$$

The main signals along the two main arms of the interferometer are

$$s_i(t) = \nu_k t - \nu_k l_i - \nu_i t + p_k(t - l_i) - p_i(t), \qquad \mathrm{II.3}a$$

$$s_k(t) = \nu_i t - \nu_i l_i - \nu_k t + p_i(t - l_i) - p_k(t), \qquad \mathrm{II.3}b$$

where i is chosen from the set {1, 2} and k is chosen appropriately from the set {3, 4}. By combining the signals from the two ends of each arm, one forms an effective two-way "Doppler" signal for each arm

$$z_i(t) = s_i(t) + s_k(t - l_i) = -2\nu_i v_i t + p_i(t - 2l_i) - p_i(t), \qquad \mathrm{II.4}$$

where $v_i = dl_i/dt$. In the unequal-arm algorithm, it is assumed that the velocity signal one is trying to see is small compared to the phase noise, or at least that it is small within the bandwidth where one is trying to detect it. Then one of the arms, say $z_1(t)$, can be used to determine p_1 (t) and to form $p_2(t) = p_1(t) + \sigma(t)$. From this knowledge, the phase noise expected in the differenced signal

$$\delta(t) \equiv z_1(t) - z_2(t) \qquad \text{II.5}$$

can be predicted and subtracted away to give a signal that is free of phase noise from the lasers.

In Reference 1, it was assumed that the readout of the phase in each receiver (Eq. 11.3) was limited by shot noise only. However, in the particular application that drove the development of the unequal-arm algorithm in the first place the detection of 10^{-3} Hz gravitational waves in a Michelson interferometer formed from four free-flying spacecraft there will generally be a very large, nearly constant Doppler rate in the data. The problem that is caused by this high fringe rate is that the absolute number of cycles that must be counted in a typical 1000 s sample time will be very large (as much as 5 X 10^{10} cycles for the 50 MHz fringe rate produced by a 50 m/s relative velocity) and that this count must be resolved to the ultimate precision required for the post-processed interferometer, $\sim 10^{-5}$ cycles. This translates into a frequency standard stability of $\sim 10^{-16}$ at 1000s, a requirement beyond the capability of any known space frequency standards.

It is the purpose of the rest of this paper to describe a way in which this requirement can be circumvented. Essentially, the method uses the fact that, over the time scales of interest, the armlength of the interferometer represents the most stable delay line ever created. One may therefore use this delay line to compare the frequency standard with itself and stabilize it, or, what is equivalent, to compare the frequency standards at the two ends of the arm with each other and correct for the noise they introduce. Two hardware realizations of this method can be envisioned. in one, there is a frequency standard on each spacecraft in addition to the main laser, and the outgoing laser signal is modulated at the frequency of the RF standard. In the other, there is a second, lower-power laser on board each spacecraft, the two lasers being locked to successive linear modes of the same Fabry-Perot stabilization cavity. The superposition of the two laser signals in the outgoing beam provides the modulation of the transmitted signal, while the beat frequency between the two stabilized lasers, read out on a fast photodiode, is the local RF frequency standard. The simplicity of the latter scheme has much to recommend it and will be the scheme that we will discuss here.

111. Transmitter

The laser transmitter system is shown in Figure 2. The heart of the system is the primary transmitting laser of frequency ν_3 producing 1 W of 1 μ wavelength infrared signal. This laser is frequency locked to a Fabry-Perot cavity on the optical bench, providing stability at a relative level of 10^{-11} on time scales of 10s 1000s. The secondary laser, of frequency ν_3' and power 100 mW, is locked to a nearby linear mode of the same cavity, so that its frequency will be stable to the same relative accuracy and will be related to the primary laser frequency by

$$\nu_3' = \nu_3 \left(1 + \frac{m}{M}\right), \qquad \text{111.1}$$

where m is a small integer and M is the integral number of wavelengths of ν_3 within the cavity length. For a 10 cm cavity, M *will* be of order 10^5, so that the frequency offset between ν_3 and ν_3' will be of order

$$f_3 \equiv \nu_3' - \nu_3 = 10^{-5} m \nu_3 = m(3 \times 10^9)\,\mathrm{Hz}. \qquad \text{III.2}$$

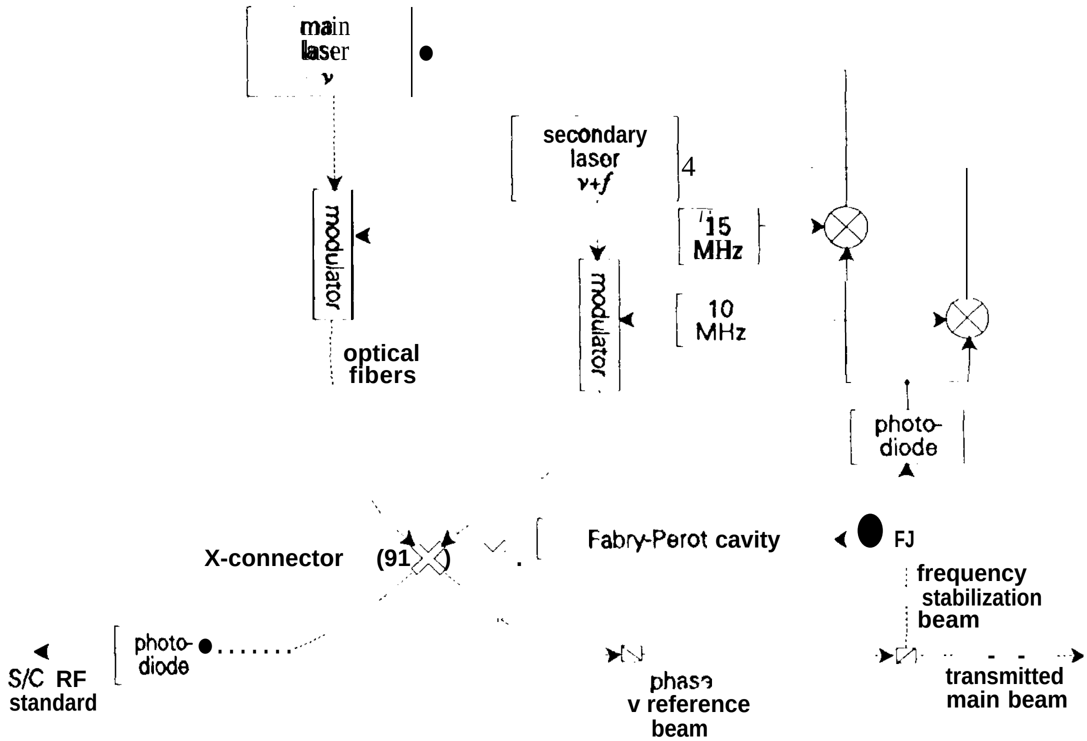

Figure 2. Laser transmitter block diagram.

The signals from the two lasers are mixed in a 9:1 coupler. Here 0.9 W from the primary laser and 0.01 W from the secondary laser are combined to go out onto the optical bench, while 0.1 W of primary laser power will be mixed with 0.09 W of secondary power to provide a nearly completely modulated RF signal at f_3 in the output of the frequency standard photodiode. Experience with such RF standards, formed from two lasers locked to the same cavity, shows that RF stability of a few parts in 10^{15} may be expected at 1000 s sample times in an RF frequency of 10 GHz. This frequency standard serves for all critical timing in the tracking system, notably for the phase measurement of the received signal (see Section IV).

The phases of the two signals sent onto the optical bench are given by

$$\phi_3(t) = \nu_3 t + p_3(t) \qquad \phi_3'(t) = \nu_3' t + p_3(t) + q_3(t) \qquad \text{III.3}$$

where the laser phase noise in the primary laser is $p_3(t)$ and the noise in the secondary laser,

$p_3'(t)$, has been written in terms of the phase noise $q_3(t) \equiv p_3'(t) - p_3(t)$ in the derived RF standard.

IV. Receiver

The receiver block diagram is shown in Figure 3. Light from the far spacecraft is received and mixed with a portion of the local laser to generate beat frequencies that are to be tracked and read out in the photodiode. The photodiode output will contain both the Doppler frequency D and the offset frequency f between the primary and secondary laser frequencies. Let us first look at the details of the signal acquisition.

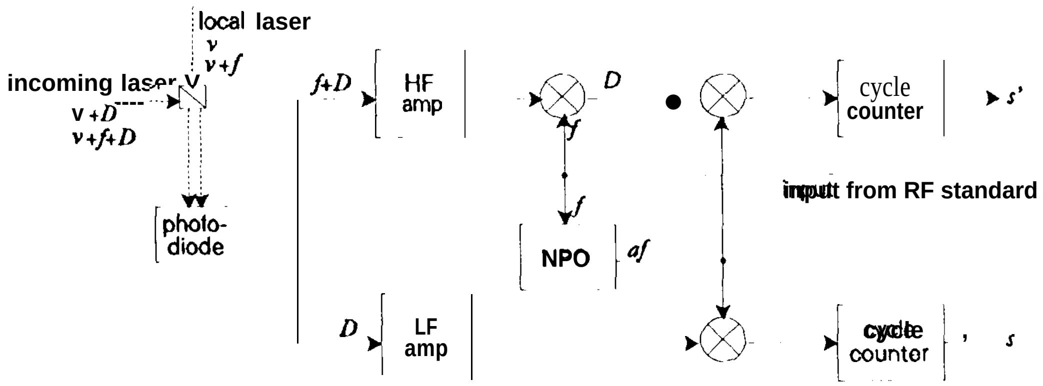

Figure 3. Laser receiver block diagram. $D\,(=\nu v)$ is the Doppler frequency shift.

The two incoming signals are given by

$$\begin{aligned} \phi_3(t-l_1) &= \nu_3 t - \nu_3 l_1 + p_3(t-l_1) \\ \phi_3'(t-l_1) &= \nu_3' t - \nu_3' l_1 + p_3(t-l_1) + q_3(t-l_1) \end{aligned} \qquad \text{IV.1}$$

where l_1 is the one-way light time which will be composed of an initial l_0 plus slow changes due to the velocities of the two spacecraft, After expanding $l_1 = l_0 + v_1 t$ and dropping the constant phase term involving l_0, Eq. IV.] becomes

$$\begin{aligned} \phi_3(t-l_1) &= (1-v_1)\nu_3 t + p_3(t-l_1) \\ \phi_3'(t-l_1) &= (1-v_1)\nu_3' t + p_3(t-l_1) + q_3(t-l_1) \end{aligned} \qquad \text{IV.2}$$

where the Doppler frequency shift is now explicit.

The two signals ϕ_3 and ϕ_3' will be mixed on the optical bench with the small fraction of the outgoing transmitted signals

$$\phi_1(t) = \nu_1 t + p_1(t) \qquad \phi_1'(t) = \nu_1' t + p_1(t) + q_1(t) \qquad \text{IV.3}$$

The instantaneous field on the photodetector will then be

$$E(t) = E_3 \sin\phi_3(t - l_1) + E_3' \sin\phi_3'(t - l_1) + E_1 \sin\phi_1(t) + E_1' \sin\phi_1'(t), \qquad \text{IV.4}$$

where E_i is proportional to the square root of the power in each signal. The intensity of the signal will be

$$\begin{aligned} I(t) \propto E^2(t) =& E_1E_1' \sin(\phi_1 - \phi_1') + E_1E_3 \sin(\phi_1 - \phi_3) + E_1E_3' \sin(\phi_1 - \phi_3') \\ &+ E_1'E_3 \sin(\phi_1' - \phi_3) + E_1'E_3' \sin(\phi_1' - \phi_3') + E_3E_3' \sin(\phi_3 - \phi_3'), \end{aligned} \qquad \text{IV.5}$$

where the time arguments of the ϕ_i have been dropped for simplicity. Terms at optical frequencies have been dropped in Eq. IV.5 since they will be too high to appear in the output of the photodiode.

Let us expand the arguments of the sine functions for each of the terms in Eq. IV.5, labeling them by their amplitudes:

$$\begin{aligned} E_1E_1' &: \quad f_1 t + q_1(t) \\ E_1E_3 &: \quad (\nu_3 - \nu_1)t - v_1\nu_3 t + p_3(t - l_1) - p_1(t) \\ E_1E_3' &: \quad (\nu_3 - \nu_1)t - v_1\nu_3 t + f_3 t - v_1 f_3 t + p_3(t - l_1) - p_1(t) + q_3(t - l_1) \\ E_1'E_3 &: \quad (\nu_3 - \nu_1)t - v_1\nu_3 t - f_1(t) + p_3(t - l_1) - p_1(t) - q_1(t) \\ E_1'E_3' &: \quad (\nu_3 - \nu_1)t - v_1\nu_3 t + (f_3 - f_1)t - v_1 f_3 t + p_3(t - l_1) - p_1(t) + q_3(t - l_1) - q_1(t) \\ E_3E_3' &: \quad f_3 t - v_1 f_3 t + q_3(t - l_1) \end{aligned} \qquad \text{IV.6}$$

Now the primary signal we wish to track in order to determine the variation in the armlength is the E_1E_3 term, which we have called $s_1(t)$

$$s_1(t) = (\nu_3 - \nu_1)t - v_1\nu_3 t + p_3(t - l_1) - p_1(t) \qquad \text{IV.7}$$

It will contain only a constant count rate $(\nu_3 - \nu_1)t$ and laser phase noises $p_3(t - l) - p_1(t)$ in addition to the Doppler signal $v_1\nu_3 t$. As was discussed in Section 11, the problem in reading out this signal is that, even if $\nu_3 = \nu_1$, the Doppler rate can give a large enough frequency that the local frequency standard will lack the stability to determine its phase at a level of accuracy consistent with the ultimate interferometer accuracy requirement. However, these errors may be estimated and corrected if the third term, the E_1E_3' term, is beat against the local frequency reference $f_1 t + q_1(t)$ to give a phase signal

$$s_1'(t) = (\nu_3 - \nu_1)t - v_1\nu_3 t + (f_3 - f_1)t + v_1 f_3 t + p_3(t - l_1) - p_1(t) + q_3(t - l_1) - q_1(t) \qquad \text{IV.8}$$

Then the difference $r(t) \equiv s'_1(t) - s_1(t)$ will contain only small nearly constant frequencies plus the data on the frequency standard noise $q(t)$:

$$r(t) = (f_3 - f_1)t - v_1 f_3 t + q_3(t - l_1) - q_1(t) \qquad \text{IV.9}$$

There is, however, a problem in reading out $s'_1(t)$. This is that the $E_1 E'_3$ term we need could have nearly the same frequency as the $E'_1 E_3$ term. The two frequencies are

$$\nu_3 - \nu_1 + v_1\nu_3 + f_3 - v_1 f_3 \quad \text{and} \quad \nu_3 - \nu_1 - v_1\nu_3 + f_1. \qquad \text{IV.10}$$

If f_1 and f_3 are very close, then these two frequencies will only be separated by the Doppler $v_1 f_3$, which will be close to zero when the relative radial velocity is small, in which case the two terms will superimpose and corrupt the measurement of $s'_1(t)$. On the other hand, if tile f_1 and f_3 frequencies are too far apart, then the $f_3 - f_1$ signal in Eq. IV.9 will not be low enough to be read out itself without introducing; phase errors due to the instability of the RF standard. The ideal separation between f_1 and f_3 would be a few kilohertz. At these frequencies a normal 10^{-11} oscillator would easily be able to count accurately at the 10`5 rad level, while the $s'_1(t)$ signal could still be easily resolved from the unwanted frequency at $\nu_3 - \nu_1 - v_1\nu_3 t + f_1$.

Of course, the frequencies f_1 and f_3 are not freely specifiable. They are related to the lengths of the Fabry-Perot cavities on the two spacecraft because they are a fraction m/M of whatever the primary laser frequency is set to be. '1'bus, if the difference between ν_3 and ν_1 is too small, then f_3 will be too close to f_1 to resolve f_3 in the signal tracking. IIow'ever, it is difficult to machine the two laser cavities so that they will resonate at nearly the same frequencies anyw ay. indeed, a difficult but feasible requirement for the tolerance required in the cavity lengths is 0.1 μm, corresponding to a frequency uncertainty from one spacecraft to the other of about 300 Mhz. '1'bus, a frequency offset of this order of mag nitude will occur naturally, without any special effort to separate them, and the difl'e.rent.e $(f_3 - f_1)$ would then naturally be about 3 KHz, in line with the requirements.

V. Signal Analysis and Noise

As may be seen by reference to Figure 3, the two signals $s_1(t)$ (Eq. IV.7) and $s'_1(t)$ (Eq. IV.8), both at frequencies near $\nu_3 - \nu_1 + v_1\nu_3$, are first reduced to countable frequencies by beating with the output from a numerically programmed oscillator (NPO) that is referenced to the RF frequency standard $f_1 + q_1(t)$, where $q_1(t)$ is the noise in the receiver's RF frequency standard photodiode output. The NPO will divide f_1 by the appropriate ratio to give a frequency $af_1 = \nu_3 - \nu_1 + v_1\nu_3$. The output of the mixer will be at a frequency of a few kilohertz which can be easily counted to 10^{-5} cycle accuracy. The noise in the photodiode output will be dominated by shot noise n(t) and $n'(t)$ in the two amplified signal s. In addition, the RF signal from the NPO will contribute noise proportional to the noise in the RF standard. The final expressions for the two measured signals, including noise, will then be

$$s_1(t) = (\nu_3 - \nu_1)t - v_1\nu_3 t + p_3(t - l_1) - p_1(t) - aq_1(t) + n_1(t) \qquad \text{V.1}$$

and

$$\begin{aligned} s_1'(t) = {} & (\nu_3 - \nu_1)t - (f_3 - f_1)t - (\nu_3 + f_3)v_1 t \\ & + p_3(t - l_1) - p_1(t) + q_3(t - l_1) - (a+1)q_1(t) + n_1'(t) \end{aligned} \qquad \text{V.2}$$

and the RF error signal will be

$$r_1(t) \equiv s'(t) - s(t) = (f_3 - f_1)t - v_1 f_3 t + q_3(t - l_1) - q_1(t) + n_1'(t) - n_1(t) \qquad \text{V.3}$$

At the same time, S/C **3** will receive the signal generated by S/C 1 with identical hardware and will form

$$s_3(t) = (\nu_1 - \nu_3)t - v_1\nu_1 t + p_1(t - l_1) - p_3(t) - aq_3(t) + n_3(t) \qquad \text{V.4}$$

and

$$r_3(t) = (f_1 - f_3)t - v_1 f_1 t + q_1(t - l_1) - q_3(t) + n_3'(t) - n_3(t) \qquad \text{V.5}$$

The a in S/C 3 will be determined independently from that of S/C 1, but they will be very close, since they are reading out nearly the same main counting frequency with nearly the same RF frequencies.

The signals, s_i and r_i, will be telemetered to the ground where in software one may form a "Doppler" signal for the link (see Eq. 11.4)

$$\begin{aligned} z_1(t) = {} & 2(\nu_3 - \nu_1)t - (\nu_3 + \nu_1)v_1 t + p_1(t - 2l_1) - p_1(t) \\ & - a[q_3(t - l_1) + q_1(t)] - n_1(t) + n_3(t - l_1) \end{aligned} \qquad \text{V.6}$$

along with two clock self-monitoring signals

$$x_1(t) = 2(f_1 - f_3)t - (f_1 + f_3)v_1 t + q_1(t - 2l_1) - q_1(t) + n_1'(t) - n_1(t) + n_3'(t - l_1) - n_3(t - l_1) \qquad \text{V.7a}$$

$$x_3(t) = 2(f_3 - f_1)t - (f_1 + f_3)v_1 t + q_3(t - 2l_1) - q_3(t) + n_3'(t) - n_3(t) + n_1'(t - l_1) - n_1(t - l_1) \qquad \text{V.7b}$$

In these expressions, constant terms and terms quadratic in v_1 have been dropped. The clock correction procedure' begins with these two signals. The constant rate $f_3 - f_1$ and the slow variations due to v_1 are first fit out to give a high-passed version $\hat{x}_i(t)$ of each of the $x_i(t)$. These are then Fourier transformed and deconvolved with the inverse transfer function for the differencing at $t = 2l_1$ to give

$$\hat{q}_i(\omega) = \frac{\tilde{x}_i(\omega)}{1 - e^{2i\omega l_1}} \approx q_i(\omega) + \frac{n_3'(\omega) - n_3(\omega) + n_1'(\omega) - n_1(\omega)}{2i\omega l_1} \qquad \text{V.8}$$

where $\hat{q}_i(\omega)$ is the estimated value of $q_i(\omega)$. The last term, taken in the low frequency limit $\omega << 1/l_1$, shows the shot noise limit to this determination. The Fourier reconstructed time series for the clock noise is then

$$\hat{q}_i(t) = q_i(t) + \frac{n_3' + n_1'}{4\pi f l} \qquad \text{V.10}$$

in which we have assumed that the shot noise is stationary and that the n' shot noise will dominate since the laser at ν' is at lower power than tile laser at ν.

Since the clock noise contribution to $z_1(t)$ is $a[q_3(t - l_1) - q_1(t)]$, this noise can now be estimated and subtracted away, giving

$$\begin{aligned} \hat{z}_1(t) &\equiv z_1(t) + a[\hat{q}_3(t - l_1) - \hat{q}_1(t)] \\ &= (\nu_3 - \nu_1)t - (\nu_3 + \nu_1)v_1 t + p_1(t - 2l_1) - p_1(t) \\ &\quad + n_1 + n_3 + \frac{a}{2\omega l_1}(n_1' + n_3') \end{aligned} \qquad \text{V.12}$$

This signal now contains only the constant count rates, the laser phase jitter which will be canceled by the interferometer algorithm, the unavoidable shot noise n_1 and n_2 in the detection of the main laser, and the greatly reduced effect of shot noise from the secondary laser. As may be seen, the procedure we have followed, using the telemetered signals $s_1(t)$, $r_1(t)$, $s_3(t)$, and $r_3(t)$, as defined in Eqs. IV.1 5, has succeeded in suppressing the unwanted clock noise and replacing it by only a fraction $a/\omega l_1$ of the larger shot noise in the secondary laser measurement. This noise will rise at low Fourier frequency as w-l, as shown in Eq. V.1O, but the fraction a, equal to the ratio between the count frequency $\nu_3 - \nu_1 - v_1\nu_3$ and the RF frequency f, can be rather small. For a Doppler frequency or frequency offset between the two main lasers of 300 MHz and a RF frequency of 10 GHz, the value of a will be 0.03.

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References

1. G. Giampieri, R.W. Hellings, M. Tinto, P.L. Bender, and J.E. Failer, Physical Review D, submitted.